Acta Technica 62 No. 1A/2017, 225–236 © 2017 Institute of Thermomechanics CAS, v.v.i.

Application research on an extended fractional lower-order cyclic music algorithm in radio frequency narrow band signal time delay estimation in wireless positioning

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Abstract. In view of the problems of wireless location technology for the smooth transmission of signals in the steady state and DOA estimation, this paper proposed an extended fractional lower order cyclic MUSIC algorithm which is based on the extended helical circular array algorithm. The principle is that the spiral and conjugate are jointly related. In view of the error estimation of the intermediate frequency difference in the original MUSIC algorithm, the method of MUSIC frequency detection time delay estimation based on frequency difference compensation is proposed. The advantage of high resolution of the original MUSIC algorithm was fully played, and at the same time the accuracy of the estimation of the time difference was improved, which made up for the deficiencies of the original MUSIC algorithm. In the simulation experiment, it is concluded that the improved algorithm is better than the traditional algorithm. This paper not only provides a new idea for wireless positioning technology, but also promotes the development of the wireless industry.

Key words. Fractional lower order cyclic MUSIC algorithm, wireless positioning, radio frequency narrow band signal, time delay estimation.

1. Introduction

Estimation of time difference is the most important point in wireless location technology. It has characteristics of high positioning accuracy, long effective time, small external interference, and small probability of being found; therefore, it has been widely used in a variety of engineering [1, 2]. The main reason for the existed estimation error of time difference is that two receivers are actually independent. The two independent receivers cannot cooperate with each other; therefore, amplitude

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cannot match each other as well. This error will cause a serious deviation of the signal on both sides, so as to affect the time difference estimation of the positioning signal. This effect has not been considered in the existing algorithms [1–4]. In view of the existing problems of time difference estimation, this paper presents an extension of fractional lower order cyclic MUSIC algorithm by using the method of frequency difference detection to compensate the time difference. Therefore, the advantage of high resolution of the common MUSIC algorithm can play a full role in improving the accuracy of the estimation of the time difference [5].

2. State of the art

Using statistics to establish mathematical model, and the tool is the stable distribution of α. The stable distribution method of α is based on the concept of limit distribution. This new distribution is more popular. This distribution will become more stable if the constraint of this kind of distribution law is enlarged in a certain way. Based on this, the paper thought under the constraint of limit distribution, it is very suitable for many situations. When under some specific circumstances, even if it is beyond the constraints, it still can be used.

Gauss distribution is a kind of special distribution mode, which is approximately the same as the above mentioned distribution of α [6]. When the α distribution is specifically required, Gauss distribution will be produced. Obviously, the range of Gauss distribution is narrower. But the two are not much different in terms of stability. According to the research on Gauss distribution, it can be deduced to the α stable distribution, which is absolutely applicable. Therefore, the random variables are arranged and input into a certain system according to the α stable distribution. The results are identical to the original input variables in their own nature. The α stable distribution has a very important role in the processing of positioning signals. It will be the focus of the next step. Because the distribution of the noise signal generated in this distribution mode is very similar to that of the actual operation, it can be a good way to simulate the generation and propagation of noise generated in the process as well as to set and save the relevant data to record more details of all kinds of noise signals.

In recent years, as an important part of signal processing, time delay estimation method has been widely used in radar, sonar, wireless communication and other fields [7]. Based on the classical estimation method of time delay, due to the limitation of signal bandwidth, the estimation accuracy will decrease obviously when the signal source distance is close, therefore, it is impossible to separate similar signals, which cannot meet the needs of practical applications in many fields. In order to solve the problem of delay estimation in close distance signal source, Schmidt and other scholars [8] have proposed time domain frequency domain estimation methods based on subspace, however, these methods have special requirements for the array flow patterns, and the time delay estimation problem is transformed into a sinusoidal parameter estimation problem. Pallas and Hasan [9–10] have proposed time delay estimation methods with high resolution, however, these methods are only suitable for wideband or flat spectrum signals, and these methods do not take into account the influence of intermediate frequency deviation on the estimation accuracy.

The envelope extraction method based on Hilbert transform which can eliminate the deviation directly without the estimation of intermediate frequency deviation has been introduced in literature [11]. There are many methods to estimate the intermediate frequency deviation, such as square frequency doubling method, frequency domain correlation method and so on. A circular MUSIC algorithm based on uniform circular array has been proposed in literature, which is called B-Cyclic-MUSIC algorithm; although the B-Cyclic-MUSIC algorithm can well estimate the azimuth and elevation angle of the incoming signal, the algorithm is not a robust algorithm, that is, the algorithm is effective only in the Gauss noise conditions, while the performance of the algorithm will be degraded when there is impulsive noise in the environment. Based on this, this paper proposed the UCA-FLOCC-MUSIC algorithm based on uniform circular array. The proposed method can not only suppress the noise and interference of a stable distribution, but also be effective in Gauss noise condition, which is a robust DOA estimation algorithm with high resolution.

3. Methodology

3.1. Fractional lower order cyclic correlation and fractional lower order conjugate cyclic correlation

When the array is a uniform matrix, the rank of the matrix is M. When the position signal is incident into the array, the bandwidth of the signal is narrow band, and its rank is L. The distance between the array elements in each matrix is K_a, and the remaining signals are not in accordance with the predetermined frequency to disturb signals circularly. The complex envelope signals of the output signal of the first k element can be expressed as

$$x_k(t) = \sum_{m=1}^{K_a} A_{km} s_m(t) + n_k(t),\ k = 1, 2, \cdots, m\,. \tag{1}$$

In the formula, the response of the first k array elements of the M signal can be expressed as A_{km}, $S_m(t)$ is the signal, independent and identically distributed complex noise can be expressed as $n_k(t)$. The observed signal vector may be expressed by the matrix equation

$$X(t) = A(\theta)S(t) + N(t)\,. \tag{2}$$

Here, the observed signal vector is expressed as $X(t) = [x_1(t), \cdots, x_m(t)]^{\mathrm{T}}$, and specified cycle frequency is the incident signal vector of ε, which can be expressed as $S(t) = [s_1(t), s_2(t), \cdots, s_{K_a}(t)]^{\mathrm{T}}$. When $K_a < L$, $A(\theta) = \{A_{km}\}_{M\times K_a} = [a(\theta_1),\ a(\theta_2), \cdots,\ a(\theta_{K_a})]$. Besides, $a(\theta_m) = \left[1, \mathrm{e}^{-j\omega_m}, \cdots, \mathrm{e}^{-\mathrm{j}(M-1)\omega_m}\right]^{\mathrm{T}}$, $\omega_m = 2\pi\frac{d}{\lambda}\sin(\theta_m)$, the incidence angle is θ_m, d is the distance, λ is the wavelength. Interference and noise vectors received by the array can be expressed as $N(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^{\mathrm{T}}$, and the noise obeys the stable distribution of α.

The correlation is described by the following two equations:

$$R^P_{XX}(\varepsilon;\tau) = \left\langle X(t+\tau/2)\left[X^H(t-\tau/2)\right]^{\langle p-1\rangle} \mathrm{e}^{-\mathrm{j}2\pi\varepsilon t}\right\rangle_t , \tag{3}$$

where H is used as the conjugate transpose. Symbol ε is used as the cyclic frequency, and p is the order of fractional lower order cyclic statistics. Then fractional lower order cyclic correlation matrix and element of (i,k) are given by the following equation.

$$R^P_{X_iX_k}(\varepsilon;\tau) = \left\langle x_t(t+\tau/2)\left[x^*_k(t-\tau/2)\right]^{\langle p-1\rangle} \mathrm{e}^{-\mathrm{j}2\pi\varepsilon t}\right\rangle_t . \tag{4}$$

3.2. Frequency detection time delay estimation model

The positioning signal received at the receiving end is expressed as $x(n)$ and$y(n)$, and the duration time of the positioning signal is $s(n)$. The radio frequency of the positioning signal of the narrow band is T_r, the time difference between the two ends of the signal is t_0, and f_s expresses signal sampling rate. When the frequency error is 0, then the model of time difference can be expressed as

$$\begin{cases} x(n) = s(n) + w_1(n), \\ y(n) = \beta s(n-D_t) + w_2(n). \end{cases} \tag{5}$$

In the formula, we use $w_1(n)$ and $w_2(n)$ to indicate noise which is unrelated with signal source, and discrete time delay of $x(n)$ and $y(n)$ is $D_t = t_0 f_s$. We use β to the express signal attenuation factor. $n = 0, 1, \cdots, K_r - 1$ and $K_r = T_r f_s$ are sampling points of the signal. Complement and illustrate the above equations to obtain the equation

$$R_{xy}(m) = \sum_{n=0}^{N-1} x(n)y(n+m) , \tag{6}$$

where $S(k)$ and $W_1(k)$ are assumed to be the result of the transformation of the signal $s(n)$, and the noise $w_1(n)$, and the mutual power spectrum density is $x_p(k) = DFT(R_{xy}(m))$. According to the above density relationship, the density can be expressed by XY:

$$x_p(k) = X(k)Y^*(k) . \tag{7}$$

In the formula (7),

$$X(k) = DFT\left[s(n) + w_1(n)\right] = S(k) + W_1(k) , \tag{8}$$

$$Y^*(k) = DFT^*\left[\lambda s(n-D_t) + w_2(n)\right] = \lambda S^*(k)\mathrm{e}^{\mathrm{j}2\pi k D_t/N} + W^*_2(k) . \tag{9}$$

The above formula is substituted uniformly.

$$x_p(k) = \lambda\left|S(k)\right|^2 \mathrm{e}^{\mathrm{j}2\pi k D_t/N} + W(k),\ k = 0, 1, \cdots, N-1 \tag{10}$$

Based on $W(k) = \lambda S^*(k)e^{j2\pi k D_t/N} W_1(k) + S(k)W_2^*(k) + W_1(k)W_2^*(k)$, time difference estimation problem can be transformed into a spectral wave frequency estimation. The above equation is also converted similarly:

$$X_p = \lambda \wedge S + W \tag{11}$$

In the formula

$$X = \begin{bmatrix} X(0) & X(1) & \cdots & X(n-1) \end{bmatrix}^T \tag{12}$$

$$\wedge = diag\begin{pmatrix} 1 & e^{j\omega} & \cdots & e^{j(N-1)\omega} \end{pmatrix}. \tag{13}$$

In the formula, $\omega = 2\pi D_t/N$, In order to optimize the test efficiency of the new algorithm and the estimation accuracy of the time difference, the above data X_p an be segmented, so as to divide the whole data into $L = N - M + 1$ submatrix.

For positioning signal in such a narrow band in transmitter, when the pulse signal is concentrated in a range of frequency, the value of L will not be very large. The estimated value $\hat{\omega}_1$ of angular frequency is obtained by constructing spatial spectral function; thus, the time delay value is obtained as

$$\hat{D}_t = \hat{t}_0 f_s = -\hat{\omega}_1 N/(2\pi). \tag{14}$$

3.3. Time delay estimation method based on envelope extraction

The literature mentioned a method using Hilbert transformation to directly eliminate the difference produced by all the intermediate frequency instead of estimating again. The positioning signal model based on this kind of conversion is

$$\begin{cases} x(n) = a(n)\cos(\omega_0 n + \varphi_1(n)) + w_1(n) \\ y(n) = a(n - D_t)\cos((\omega_0 + \Delta\omega_0)n + \varphi_2(n)) + w_2(n). \end{cases} \tag{15}$$

In formula (15), $a(n)$ is the signal amplitude, $\varphi_1(n)$ and $\varphi_2(n)$ are the initial phases. We can see that the acquisition of time delayD_t is actually through $a(n)$ and $a(n - D_t)$, then we can offset the error value of the intermediate frequency after we obtain D_t.

The following formula can be used in practical use:

$$\hat{a}(n) = \sqrt{x^2(n) + H^2[x(n)]}, \tag{16}$$

$$\hat{a}(n - D_t) = \sqrt{y^2(n) + H^2[y(n)]}. \tag{17}$$

Here, H [] represents the transformation of Hilbert. By summing up all the above formula we can be found that this transformation will have a huge impact on the noise. Moreover, this impact can make the algorithm have a great effect on the accuracy of the estimation of time difference.

3.4. Time delay estimation method based on frequency difference compensation

When the above solutions are unable to reasonably eliminate the error caused by the intermediate frequency, we can first calculate and estimate the error values, and then make up for the signal. The value we make up for is the above estimation value. Set the estimation value of intermediate difference Δf_0 as $\Delta \hat{f}_0$, we compensate for the received signal $x(n)$:

$$x_0(n) = x(n)\mathrm{e}^{\mathrm{j}2\pi n \Delta \hat{f}_0 / f_s}\,. \tag{18}$$

In the same way, the estimated value $\hat{\omega}_t = -2\pi \hat{D}_t / N$ of the angular frequency is obtained by using the MUSIC algorithm, then the corresponding time delay estimation is $\hat{D}_t = -\hat{\omega}_t N/(2\pi)$. We assume that the error value of intermediate frequency is $\Delta f = \Delta \hat{f}_0 - \Delta f_0$, and then the corresponding experimental estimation error will be

$$\varepsilon_t(k) = \frac{N^2}{2\pi f_s}\frac{d^2\varphi(k)}{dk^2}\Delta f\,. \tag{19}$$

4. Result analysis and discussion

4.1. Robustness analysis

In this paper, simulation experiments are used to verify the performance and robustness of the algorithm. The above three algorithms are brought into the simulation experiment, and the experimental ontology is the uniform array of $M = 8$. The incident signal selects a narrow band signal with the same power, and all the noise distribution adopt stable distribution of α. The preset cycle frequency of the incident signal is $\varepsilon = f_b$, and the frequency of the interference signal is 4.2 MHz. For the collection of the incident signal, the selected time frequency is 30 MHz. Among the obtained results, finally we get the pros and cons of the three algorithms based on the comparison of angular resolution.

Simulation experiment 1: in this paper, the interference shielding ability of the noise in the algorithm is affected by the low order fractional matrix P. The choice of SOI is 10 degrees and 20 degrees, and the initial angle of the interference information is 50 degrees. The noise in this experiment is consistent with the stable distribution of α, and the value α of this experiment is 1.4. The generalized noise-signal ratio is respectively set to 10 dB, 15 dB and 20 dB. The number of snapshot of signals is 1000. 500 cycles of Monte Carlo Simulation is carried out in each experiment.

Figure 1 shows the curve of the mean square error changed with the fractional lower order moment P. From Fig. 1 we can detect that under different signal and noise ratio conditions, when $1 < p < \alpha$, the root mean square error of the algorithm is relatively small. Apparently, when $p = 2$, the algorithm in this paper is reduced to EX-Cyclic-MUSIC algorithm. That is to say, EX-Cyclic-MUSIC is the special case of this algorithm.

Simulation experiment 2: the effect of the snapshots used in this paper to verify the signal on algorithm. Different from experiment 1, the generalized noise-signal ratio is set to 10,dB, and the value of P will be changed to 1.2. Figure 2 is a diagram of the relationship between the RMS error and the number of signal snapshots. In this experiment, the three algorithms are using the cycle data. Cycle itself is periodic, so circulation of signal itself can be reflected only when numerical value of signal snapshots is large. Through analysis of the following figure, when the numerical value of snapshots is identical, the root mean square error of the improved algorithm is obviously smaller than that of the other two algorithms, which proved that the improved algorithm has better performance than other algorithms. With the increase of the number of snapshots, the gap between the three becomes smaller, until basically coincident.

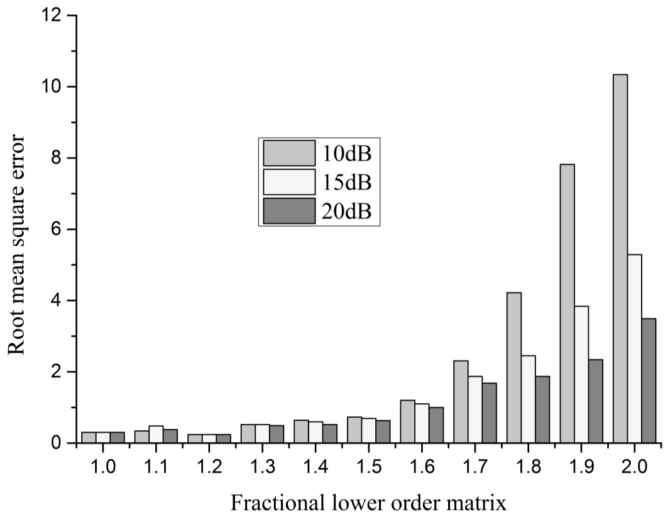

Fig. 1. RMSE of DOA estimates versus p

4.2. Time difference estimation analysis

The time difference estimation methods are compared with the simulation experiment. These three methods are MUSIC method without compensating for frequency difference, the MUSIC method with compensating for frequency difference and the method for estimating time difference by using envelope extraction. In the experiment, the acquisition time and frequency of the incident signal is 50 MHz, and the incident signal adopts modulation signal. The carrier frequency of the signal itself is 10.7 MHz. The time of acquisition of each incident signal is 20 μs. The generalized noise-signal ratio is 10 dB.

When the bandwidth of the signal is not the same, the error produced by intermediate frequency increases from 0.5 to 2.5. The signal bandwidth is 4–10 MHz every 2 minutes. 100 cycles of Monte Carlo Simulation are carried out in each experiment.

From Figs. 3–6, we can come up with the corresponding conclusion that when

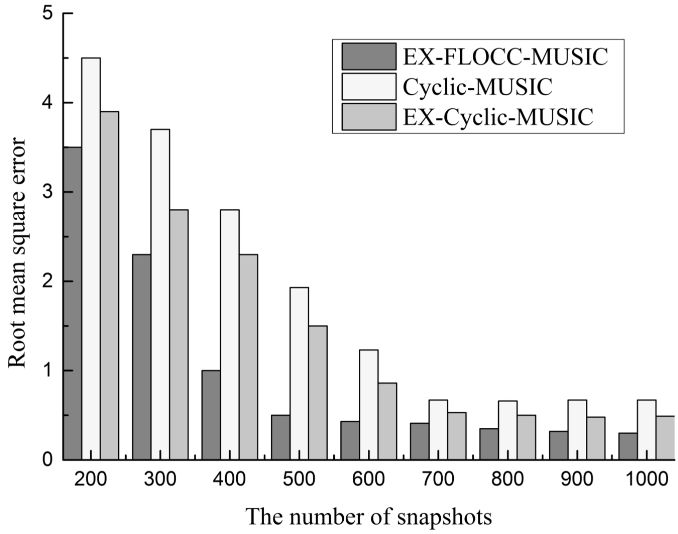

Fig. 2. RMSE of DOA estimates versus number of snapshots

the error of the intermediate frequency remains unchanged, due to the existence of error, the first estimation method can produce very large error in the estimation of time difference. The third estimation method can eliminate the error, which will not affect the accuracy of the final value. The second estimation method proposed in this paper can not only ensure the accuracy of the final results, but also produce the corresponding compensation value with the constant change of the bandwidth. Compared with the other two time difference estimation methods, this scheme achieves better results with better accuracy. When conditions are changed, it can timely generate corresponding changes, which is more suitable for the use of the current positioning systems.

5. Conclusion

This paper mainly studied how to reduce the error of the time delay estimation accuracy of radio frequency narrow band signal caused by the intermediate frequency deviation. Therefore, this paper proposed an extended fractional lower order cyclic MUSIC algorithm. The conclusions drawn are as follows:

This paper combined fractional lower order cycle and fitting spectrum correlation signal subspace, and put forward a new algorithm of MUSIC time delay estimation based on frequency difference compensation. This algorithm retains the advantages of the traditional algorithm, and it also improves the defects of the traditional algorithm. For example, it can better distinguish the angle of impulse noise, and it has good anti-interference performance. Experimental results showed that the new algorithm of MUSIC time delay estimation can effectively reduce the workload and improve the running speed. It has high precision and strong adaptability, and it is

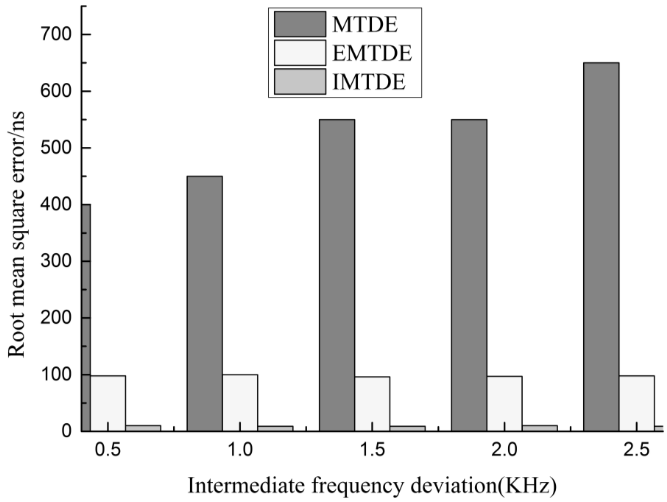

Fig. 3. Root mean square error versus intermediate frequency deviation, 10 kHz wide signal band

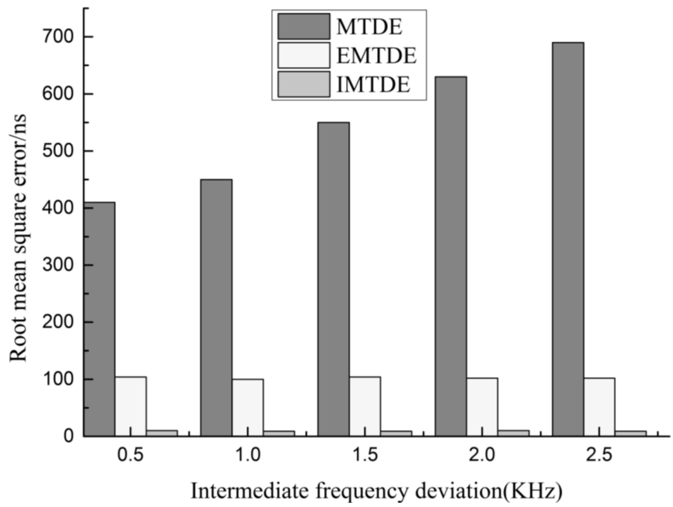

Fig. 4. Root mean square error versus intermediate frequency deviation, 8 kHz wide signal band

of great application value.

In order to solve the problem of similar signal source and time delay estimation, this paper made a comparison between the time delay estimation model and the MUSIC algorithm estimation model, and proposed an extended fractional lower order cyclic MUSIC algorithm based on frequency difference compensation. This method has high resolution and good performance in time delay estimation. Compared with the traditional algorithm, the algorithm is more adaptable and suitable for various time delay estimation conditions.

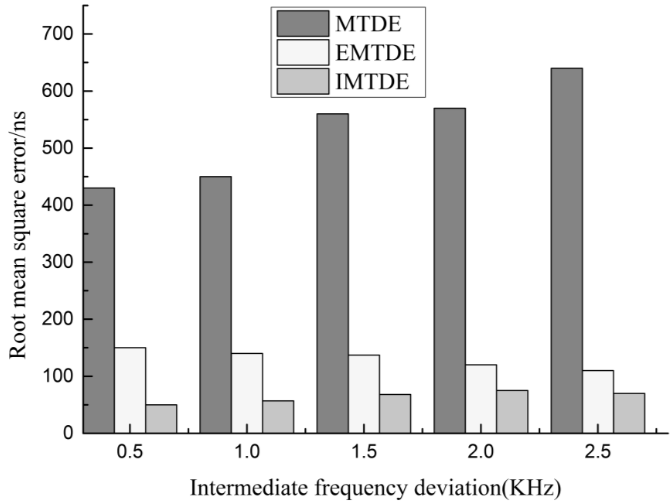

Fig. 5. Root mean square error versus intermediate frequency deviation, 6 kHz wide signal band

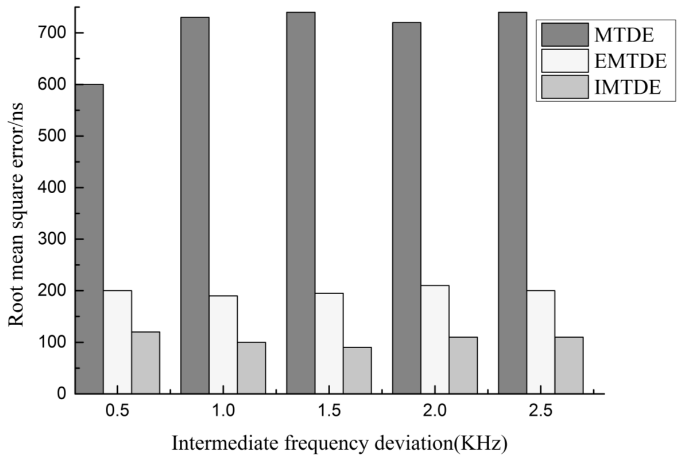

Fig. 6. Root mean square error versus intermediate frequency deviation, 4 kHz wide signal band

An extension of the fractional lower order cyclic MUSIC algorithm proposed in this paper is not comprehensive enough. This paper only focus on the deviation of intermediate frequency in time delay estimation, and there is still much research space for the analysis of other influencing factors. In the further study, we should focus on the new time delay estimation method when multiple factors coexist.

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Received May 22, 2017